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## Minimal Ten-parameter Hermitian Texture Zeroes Mass Matrices and the CKM Matrix<sup>1</sup>

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### Abstract

Hermitian mass matrices for the up and down quarks with texture zeroes but with the minimum number of parameters, ten, are investigated. We show how these *minimum parameter* forms can be obtained from a general set of hermitian matrices through weak basis transformations. For the most simple forms we show that one can derive exact and compact parametrizations of the CKM mixing matrix in terms of the elements of these mass matrices (and the quark masses).

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# 1 Introduction

Within the three-family standard model, the Yukawa interaction provides ten physical measurable parameters, the six quark masses and the four parameters of the CKM mixing matrix[1]. Although ten is certainly a large number if the model is to be viewed as a fundamental theory, this number of parameters in fact emerges from two  $3 \times 3$  Yukawa matrices which in total amount to as many as 36 parameters! Clearly there is a large number of redundant parameters. In order to better corner the mechanism of symmetry breaking it could be advantageous to work with mass matrices that exhibit only the least number of parameters, *i.e.* ten. Having to deal with a minimal set not only eases the computational task, like going from the mass matrices to the mixing matrix, but when confronting the set with the data this may even help in better exhibiting patterns of mass matrices that hint towards further relations between some of the elements of the set. In this case one can entertain the existence of symmetries and models beyond the standard model that can explain the approximate relations. At the same time this general minimum approach could also reveal whether some relations, like those that relate some mixing angles to ratios of masses, are in fact not specific to a particular constrained model but are rather generic in a much wider class of models.

Most of the *constrained* matrices that aim at relating the mixing angles to the quark masses, and hence reduce the number of parameters to much less than the needed 10, are based on so called texture zeroes mass matrices [2]. These are based either on some specific "beyond the standard model" scenario or by postulating some ad-hoc ansatz. In ansätze where these zeroes are not related to any symmetry only the non-zero elements are counted as parameters, although it is clear that there are numerous ways of keeping to ten independent parameters. For instance instead of zeroes one can take some elements to be equal or have any other definite relations between them [3, 4]. Democratic [3] mass matrices with all elements equal are a case in mind, these are a one-parameter model which when written in an appropriate basis can be turned into matrices with all elements but one being zero. In our approach we keep within the popular textures zeroes paradigm and look for those bases where only the minimum number of (non-zero) independent parameters

appears explicitly.

Obviously, dealing with less than ten parameters invariably leads to relations between masses and mixings. In many cases and for a certain range of masses the less-than-ten parameter descriptions may turn out not to be supported by data if the textures are over-predictive. On the other hand if one works with, at least, ten independent parameters then one should always reproduce the data since there should be possible to make a one-to-one mapping. A general classification of symmetric/hermitian textures zeroes with a number of parameters less than ten has been given in [2], while Branco Lavoura and Mota [5] (BLM) have been the first to point out that for non hermitian matrices some textures zero à la Fritzsch [6] were just a rewriting of the mass matrices in a special basis and thus the zeroes of the much celebrated Fritzsch ansatz were “void” of any physical content. In fact the BLM approach for non hermitian matrices still involves twelve parameters, the extra two being related to the phase conventions taken for the CKM<sup>2</sup>.

Recently an approach based on BLM has been pursued by some authors [7, 8, 9], taking a specific pattern of non hermitian matrices and in some cases re-expressing one of the mass matrices with the help of the phenomenological parametrization of the CKM matrix. In this talk we will concentrate on hermitian texture zero matrices having zeroes in the non-diagonal entries. The case with zeroes on the diagonal will be presented in a longer communication [10]. Note that we differ from [2] in that we still have ten parameters. It is known that within the standard model one can always express the mass matrices in a basis where they are hermitian [11, 5, 12]. Also, in the case of non hermitian matrices our results should be understood as applying to the hermitian square matrices,  $H = MM^\dagger$ . We supply a systematic list of all possible texture zeroes that contain the minimal set of ten parameters and show how these textures can be reached from a general set of two hermitian matrices for up and down quarks, through specific weak basis transformation which we construct explicitly. Among all the patterns that we list, one shows a particularly very simple and appealing structure which has a direct connection to the Wolfenstein parametrization [13]. For this we have been able to analytically construct

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<sup>2</sup>One should be fair and say that, sometimes, keeping one or two of the redundant parameters may prove useful. However we will stick with the minimalist description.

a compact exact formulae for the mixing matrix.

## 2 Simple Texture Zeroes Quark Mass Matrices and the Choice of Basis

The key observation as concerns the search of a suitable basis, ideally one with the maximum number of zeroes, is that starting from any set of matrices for the up and down quarks, the physics is invariant if one performs a weak basis transformation on the fields. In the case of the standard model, one can choose any right-handed basis for both the up quark fields ( $u_R$ ) and the down quark fields ( $d_R$ ), as well as any basis for the doublets of left-handed fields ( $Q_L$ ). All these bases are related to each other through unitary transformations,  $u_R \rightarrow V_u u_R$ ;  $d_R \rightarrow V_d d_R$ ;  $Q_L \rightarrow U_L Q_L$ . Therefore all sets of mass matrices related to each other through

$$M'_u = U_L^\dagger M_u V_u \quad M'_d = U_L^\dagger M_d V_d \quad (1)$$

give rise to the same physics (same masses and mixing angles in the charged current).

For hermitian matrices this means that weak basis transformations involve only a single unitary transformation, *i.e.*,  $U_L = V_u = V_d = U$  and therefore one can use either the set  $M_u, M_d$  or the set  $M'_u, M'_d$  with  $M'_f = U^\dagger M_f U$ . In the case of hermitian matrices, one is starting with a set of 18 parameters and the task is to find a unitary matrix  $U$  which can absorb 8 redundant parameters. This should always be possible since a  $3 \times 3$  unitary matrix has nine real parameters, but since an overall phase transformation  $U = e^{i\phi} \mathbf{1}$  does not affect weak bases transformations, a unitary matrix provides the required number of variables to absorb the redundant parameters.

### 2.1 Phase transformations

One special case of this type of unitary transformations which always proves useful, even in the case of non-hermitian matrices, is the one provided by unitary phase transformations

$U_{ij} = e^{i\phi_i}\delta_{ij}$ . Because a global phase does not affect the transformation, we set  $\phi_1 = 0$  without loss of generality. This type of matrix is therefore a two-parameter matrix. Applying this type of transformation on both  $M_u, M_d$  one has the freedom to choose  $\phi_{2,3}$  such that two phases out of the six contained in the hermitian  $M_u, M_d$  can be set to zero. The only restriction is that one can not, in general, simultaneously remove the phases of both  $M_u(ij)$  and  $M_d(ij)$  (*i.e.* for the same  $(ij)$ ). In any case, two parameters, or rather phases, out of the 18 can always be removed this way.

## 2.2 The simple case of a basis where one matrix is diagonal

It is always possible to take  $U = U_d^D (U_u^D)$ , that is the unitary matrix that diagonalises the down (up) matrix. In these specific bases where one matrix is diagonal, the other, non-diagonal matrix, will then have no zero in general but 9 real parameters (of which 3 can be taken as phases in the non-diagonal entries). Applying an extra phase transformation removes two phases and therefore one does indeed end up with 3 parameters in  $M_d$  (the masses) and 7 in  $M_u$  making up a total of ten which is the minimal number.

It is worth mentioning that similar bases (where one of the matrices is diagonal) have been studied in the literature but for the case of non-hermitian matrices [7, 8, 9]. It is easy to see that one can easily recover these bases. Indeed, one can apply on our hermitian matrices, the following transformations: assuming one is starting with a diagonal  $M_u$  take  $U = V_u$  as a phase transformation or simply just the unit matrix, then it is always possible to choose  $V_d$  such that  $M_d$  turns into a non-hermitian matrix but with extra zeroes. We leave the proof and a discussion of these kind of (diagonal) bases to our longer communication [10].

	$M_u$	$M_d$		$M_u$	$M_d$
1	(1,2)	(1,3) and (2,3)	10	(1,3) and (2,3)	(1,2)
2	(1,2)	(1,2) and (2,3)	11	(1,2) and (2,3)	(1,2)
3	(1,2)	(1,2) and (1,3)	12	(1,2) and (1,3)	(1,2)
4	(1,3)	(1,3) and (2,3)	13	(1,3) and (2,3)	(1,3)
5	(1,3)	(1,2) and (2,3)	14	(1,2) and (2,3)	(1,3)
6	(1,3)	(1,2) and (1,3)	15	(1,2) and (1,3)	(1,3)
7	(2,3)	(1,3) and (2,3)	16	(1,3) and (2,3)	(2,3)
8	(2,3)	(1,2) and (2,3)	17	(1,2) and (2,3)	(2,3)
9	(2,3)	(1,2) and (1,3)	18	(1,2) and (1,3)	(2,3)

Table 1: Location of the zeroes for the 18 different forms.

### 2.3 Non trivial cases: Non diagonal matrices with no diagonal zero

In the above simple case one had three zeroes<sup>3</sup>. In fact requiring that one maintains 10 parameters, and in the case of hermitian matrices where the zeroes are set on the off-diagonal elements, three is the maximum number of zeroes. The non trivial cases are when these three zeroes are shared between the up-quark and down-quark matrices, that is one off-diagonal zero in one matrix and two off-diagonal zeroes in the other. Indeed, having more than three off-diagonal zeroes, four say, one is left with the six real parameters on the diagonals plus two complex numbers which reduce to two real numbers after a phase transformation has been applied and thus leading to only 8 real parameters. Therefore, by requiring off-diagonal zeroes the problem is rather simple: one only has to combine a matrix with one off-diagonal zero with a matrix with two off-diagonal zeroes. For each of these matrices there are three possibilities of where to put the zero. All in all, one counts 18 such possibilities or patterns. These are displayed in Table 1.

All of these combinations can in fact be classified in only two distinct cases which can

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<sup>3</sup>Since one is dealing with hermitian matrices, the number of zeroes is that contained on one side of the diagonal.

not be obtained from each other by a simple relabeling of the axes. Denoting the two arbitrary hermitian mass matrices in those bases by  $M_u = A'$  and  $M_d = B'$ , these cases are explicitly:

$$A' = \begin{pmatrix} A'_{11} & 0 & 0 \\ 0 & A'_{22} & A'_{23} \\ 0 & A'^*_{23} & A'_{33} \end{pmatrix}, B' = \begin{pmatrix} B'_{11} & 0 & B'_{13} \\ 0 & B'_{22} & B'_{23} \\ B'^*_{13} & B'^*_{23} & B'_{33} \end{pmatrix} \quad \underline{\text{case I}} \quad (2)$$

and

$$A' = \begin{pmatrix} A'_{11} & 0 & 0 \\ 0 & A'_{22} & A'_{23} \\ 0 & A'^*_{23} & A'_{33} \end{pmatrix}, B' = \begin{pmatrix} B'_{11} & B'_{12} & B'_{13} \\ B'^*_{12} & B'_{22} & 0 \\ B'^*_{13} & 0 & B'_{33} \end{pmatrix} \quad \underline{\text{case II}}. \quad (3)$$

where  $A'_{11}$  is an eigenvalue. Of course, one can exchange the role of  $A'$  and  $B'$  so that  $A' = M_d$  and  $B' = M_u$ .

To prove the existence of these bases and show how they are reached, it is easiest to first move to the basis where  $A$  is diagonal.

Denoting the eigenvalues of  $A$  by  $\lambda_i$ , ( $i = 1, 2, 3$ ) and  $A'_{11} = \lambda_1$ , we have in the eigenbasis of  $A$

$$A = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, B = \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B^*_{12} & B_{22} & B_{23} \\ B^*_{13} & B^*_{23} & B_{33} \end{pmatrix}. \quad (4)$$

The unitary matrix which leads to the form for  $A'$  in both Eq. 2 (Case I) and Eq. 3 (case II) is simply

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x_2 & x_3 \\ 0 & y_2 & y_3 \end{pmatrix} \quad (5)$$

with the complex numbers  $x_2, x_3, y_2, y_3$  subject to the orthonormality conditions. It is then trivial to find the appropriate combinations of  $x_2, x_3, y_2, y_3$  that lead to either  $B'$  in the above two cases [10]. For instance in the first case, requiring  $B'_{12} = x_2 B_{12} + x_3 B_{13} = 0$  gives the appropriate  $U$ . All other cases with two off-diagonal zeroes in one matrix and one in the other are treated in an analogous way. The proofs are obtained from case I and case II just by relabeling the indices. Of course, the case where the two quark mass

matrices make up between them only two-zeroes, being much less constrained, is always easier to construct.

### 3 CKM matrices from off-diagonal texture zeroes hermitian matrices

The advantage of texture zeroes matrices yet accommodating all the ten physical parameters is that they allow to easily express the mixing matrix solely in term of the elements describing the mass matrices. One could then work backward and use the hierarchy observed in a particular parametrization of the CKM mixing matrix, together with the hierarchy in the masses, to exhibit further correlations in the elements of the mass matrices expressed in a simple basis that already exhibits zeroes.

Recently, Rašin [14] has devised a procedure to express the CKM matrix as a function of the mass matrices in the general case where no zero element is found in neither  $M_u$  nor  $M_d$ . He uses a product of rotation matrices and phase matrices to diagonalise a general  $3 \times 3$  matrix. However, even when we require  $M_u$  to be diagonal, which is a special case of [14], we are still left with large formulae which include sines and cosines of angles for which only the tangent is explicitly known. These results do not give compact expressions for the CKM matrix elements. Only when more zeroes are imposed do the results simplify. Even with the simple textures that are displayed in Table 1, the recipe given in [14] leads to tedious and complicated formulae [10] which moreover come with an ambiguity in determining the signs of the sines and cosines. We will show that, with the textures that are displayed in Table 1, there exists a more compact way of expressing the CKM that does not make use of any sines or cosines but exhibits the masses and the elements of the mass matrices explicitly.

Each combination in Table 1 will lead to a particular parametrization of the mixing matrix. We concentrate on parametrization 14 not only to illustrate how the diagonalisation of the matrices is carried out exactly, and hence how one expresses the CKM,



but also because it leads to a parametrization of the Kobayashi-Maskawa matrix which is directly related to the Wolfenstein parametrization [13].

To achieve this, we first apply a weak basis phase transformation to the form 14, such that the only remaining phase is located in the up quark matrix. Thus one is dealing with

$$M_u = \begin{pmatrix} u & 0 & ye^{i\phi} \\ 0 & \lambda_c & 0 \\ ye^{-i\phi} & 0 & t \end{pmatrix}, \quad M_d = \begin{pmatrix} d & x & 0 \\ x & s & z \\ 0 & z & b \end{pmatrix} \quad (6)$$

Note that this parametrization allows to have as input, at the level of the mass matrices, the physical mass of the charm quark,  $\lambda_c$ . In what follows all physical masses will be denoted by  $\lambda_i$ , the index  $i$  being a flavour index.

These mass matrices are diagonalised through the following unitary matrices

$$U_u = \begin{pmatrix} \sqrt{\frac{u_t}{\lambda_{ut}}} & 0 & \sqrt{\frac{u_u}{\lambda_{ut}}}e^{i\phi} \\ 0 & 1 & 0 \\ -\sqrt{\frac{u_u}{\lambda_{ut}}}e^{-i\phi} & 0 & \sqrt{\frac{u_t}{\lambda_{ut}}} \end{pmatrix}, \quad U_d = \begin{pmatrix} \sqrt{\frac{b_d d_s d_b}{\Delta \lambda_{ds} \lambda_{db}}} & \sqrt{\frac{d_d b_s d_b}{\Delta \lambda_{ds} \lambda_{sb}}} \sigma & \sqrt{\frac{d_d d_s b_b}{\Delta \lambda_{db} \lambda_{sb}}} \\ -\sqrt{\frac{d_d b_d}{\lambda_{ds} \lambda_{db}}} & \sqrt{\frac{d_s b_s}{\lambda_{ds} \lambda_{sb}}} & \sqrt{\frac{d_b b_b}{\lambda_{db} \lambda_{sb}}} \\ \sqrt{\frac{d_d b_s b_b}{\Delta \lambda_{ds} \lambda_{db}}} & -\sqrt{\frac{b_d d_s b_b}{\Delta \lambda_{ds} \lambda_{sb}}} \sigma & \sqrt{\frac{b_d b_s d_b}{\Delta \lambda_{db} \lambda_{sb}}} \end{pmatrix} \quad (7)$$

Such that

$$U_{u,d}^\dagger M_{u,d} U_{u,d} = \begin{pmatrix} \lambda_{u,d} & 0 & 0 \\ 0 & \lambda_{c,s} & 0 \\ 0 & 0 & \lambda_{t,b} \end{pmatrix} \quad (8)$$

and

$$x_i = |x - \lambda_i| \quad (e.g. \quad u_t = |u - \lambda_t|) \quad (9)$$

$$\lambda_{ij} = |\lambda_i - \lambda_j| \quad (10)$$

$$\Delta = |b - d| \quad (11)$$

$$\sigma = \text{sign of } (b - d) \quad (12)$$

Expressing the diagonalising matrices,  $U_u, U_d$ , with the help of the physical masses keeps the expressions of these matrices very compact. As  $V_{\text{CKM}} = U_u^\dagger U_d$ , we can now

write the CKM matrix exactly:

$$V_{us} = \frac{\sigma \left( \sqrt{u_t d_d b_s d_b} + \sqrt{u_u b_d d_s b_b} e^{i\phi} \right)}{\sqrt{\Delta \lambda_{ut} \lambda_{ds} \lambda_{sb}}} \quad (13)$$

$$V_{ub} = \frac{\sqrt{u_t d_d d_s b_b} - \sqrt{u_u b_d b_s d_b} e^{i\phi}}{\sqrt{\Delta \lambda_{ut} \lambda_{db} \lambda_{sb}}} \quad (14)$$

$$V_{cd} = -\sqrt{\frac{d_d b_d}{\lambda_{ds} \lambda_{db}}} \quad (15)$$

$$V_{cb} = \sqrt{\frac{d_b b_b}{\lambda_{db} \lambda_{sb}}} \quad (16)$$

$$V_{td} = \frac{\left( \sqrt{u_u b_d d_s d_b} e^{-i\phi} + \sqrt{u_t d_d b_s b_b} \right)}{\sqrt{\Delta \lambda_{ut} \lambda_{ds} \lambda_{db}}} \quad (17)$$

$$V_{ts} = \frac{\sigma \left( \sqrt{u_u d_d b_s d_b} e^{-i\phi} - \sqrt{u_t b_d d_s b_b} \right)}{\sqrt{\Delta \lambda_{ut} \lambda_{ds} \lambda_{sb}}} \quad (18)$$

$$J \equiv \frac{\det [M_u, M_d]}{2i \lambda_{uc} \lambda_{ut} \lambda_{ct} \lambda_{ds} \lambda_{db} \lambda_{sb}} = \frac{\sqrt{u_u u_t d_d d_s d_b b_d b_s b_b}}{\lambda_{ut} \lambda_{ds} \lambda_{db} \lambda_{sb}} \sin \phi \quad (19)$$

We see that these expressions are surprisingly simple given that they come from the mass matrices. Moreover, contrary to some ansätze, this type of CKM matrix can always be made to fit the data.

Nonetheless, we are now in a position to exploit the mass hierarchies. One can take  $x_x$  as a small perturbation, which means that in fact  $\lambda_f \simeq f$  where  $f$  refers to a diagonal element. In other words this assumption amounts to requiring that the diagonal elements of the mass matrices deviate very little from their corresponding eigenvalues. We then have from eq. 15 and 16,

$$d_d \simeq |V_{cd}|^2 \lambda_s, \quad (20)$$

$$b_b \simeq |V_{cb}|^2 \lambda_b. \quad (21)$$

We also have from eq.14

$$V_{ub} \simeq \frac{1}{\sqrt{\lambda_t}} \left( \sqrt{\frac{d_d b_b}{\lambda_b}} - \sqrt{u_u} e^{i\phi} \right) \simeq -\sqrt{\frac{u_u}{\lambda_t}} e^{i\phi} \quad (22)$$

where we have made the additional assumption that the terms involving the down-quarks are quadratic in the “perturbation”  $d_d \times b_b$  compared to the term originating from the up quark matrix:  $u_u$ . This additional assumption is stronger than the previous ones since it also compares the strengths of the off-diagonal elements of the up *and* down quark matrices. In any case with these mild assumptions one can now trade  $d_d, b_b, u_u$ , *i.e.*  $d, b, u$  for the moduli of  $V_{cd}, V_{cb}$  and  $V_{ub}$  and physical masses (up to some signs).

Taking into account the size of  $d_d, b_b$  and  $u_u$ , we can now write

$$V_{\text{CKM}} \simeq \begin{pmatrix} V_{ud} & \sqrt{\frac{d_d}{\lambda_s}} & -\sqrt{\frac{u_u}{\lambda_t}} e^{i\phi} \\ -\sqrt{\frac{d_d}{\lambda_s}} & V_{cs} & \sqrt{\frac{b_b}{\lambda_b}} \\ \sqrt{\frac{d_d b_b}{\lambda_s \lambda_b}} + \sqrt{\frac{u_u}{\lambda_t}} e^{-i\phi} & -\sqrt{\frac{b_b}{\lambda_b}} & V_{tb} \end{pmatrix}, \quad (23)$$

$$J = \sqrt{\frac{u_u d_d b_b}{\lambda_s \lambda_b \lambda_t}} \sin \phi. \quad (24)$$

It is interesting to see that in this parametrization the  $V_{CKM}$  can be split into elements which originate either solely from the down-quark sector<sup>4</sup> or the up-quark sector. To recover a phenomenologically viable mixing matrix, one could thus concentrate on each sector separately. Moreover this parametrization is equivalent to the standard Wolfenstein parametrization

$$V_W = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A(1 - \rho - i\eta) & -\lambda^2 A & 1 - \mathcal{O}(\lambda^4) \end{pmatrix}. \quad (25)$$

with

$$\begin{aligned} \lambda &= \sqrt{\frac{d_d}{\lambda_s}} \\ A &= \frac{\lambda_s}{d_d} \sqrt{\frac{b_b}{\lambda_b}} \\ \rho &= -\sqrt{\frac{u_u \lambda_s \lambda_b}{\lambda_t d_d b_b}} \cos \phi \\ \eta &= \sqrt{\frac{u_u \lambda_s \lambda_b}{\lambda_t d_d b_b}} \sin \phi \end{aligned} \quad (26)$$

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<sup>4</sup>A similar observation has also been made in [15].

Asking for maximal CP violation [16] sets  $\phi = \pi/2$  and leads to  $\rho = 0$ .

From the form of the  $V_{CKM}$  matrix it is now an easy matter to find phenomenologically viable quark mass matrices. Most direct from our study is the *general* feature that if  $d = (M_d)_{11} = 0$ , then  $d_d = \lambda_d = m_d$  and therefore one has the rather successful prediction [17, 18]:  $V_{us} \simeq \lambda \simeq \sqrt{\lambda_d/\lambda_s} = \sqrt{m_d/m_s}$ . Moreover, introducing the perturbative parameter  $\epsilon \ll 1$  and with all other parameters of order 1, we may write the hierarchical matrices:

$$M_u = \lambda_t \begin{pmatrix} 0 & 0 & c \epsilon^3 e^{i\phi} \\ 0 & \lambda_c/\lambda_t & 0 \\ c \epsilon^3 e^{-i\phi} & 0 & 1 \end{pmatrix}, \quad M_d = \lambda_b \begin{pmatrix} 0 & a \epsilon^3 & 0 \\ a \epsilon^3 & \epsilon^2 & b \epsilon^2 \\ 0 & b \epsilon^2 & 1 \end{pmatrix} \quad (27)$$

This leads to  $V_{us} = \sqrt{m_d/m_s} = a \epsilon$  whereas  $|V_{ts}| = b \epsilon^2 = (b/a^2) |V_{us}|^2$  with  $(b, a \sim 1)$ . Therefore, if one identifies  $\lambda = a\epsilon$  then  $A = b/a^2$ . We could “adjust”  $a, b$  (and  $c$ ) to better fit the data. The forms in Eq. 27 bear some resemblance to those presented in [15], but note that we arrive at these forms from a rather different approach.

Also in the down sector, the ansatz reproduces the correct ratio of masses. Note also that with the ansatz for the up-quark, copied somehow on that of the down quark, we get:  $|V_{ub}| \simeq c \epsilon^3$ .

## 4 Conclusions

We have shown that without any assumption on the mass matrices apart from hermiticity, it is always possible to find a quark basis such that 3 off-diagonal elements are vanishing, allowing to diagonalise unambiguously the mass matrices and obtain the mixing matrix. The case where either  $M_u$  or  $M_d$  is diagonal (and therefore all the 6 vanishing elements are contained in one single matrix) is of special interest but leads to lengthy formulae for the CKM matrix entries. In all other cases, we arrived at compact formulae for the mixing matrix. These compact formulae that express without any approximation the  $V_{CKM}$  matrix in terms of the masses and other elements of the mass matrices can be compared to popular parametrizations of the CKM matrix. The exact forms that we find make it transparent which further assumptions one can make (*i.e* more zeroes) to simplify the

structure of the mass matrices and yet be compatible with the data. We have given one such example, and in passing we have shown how starting from the general 10 parameter bases, the mere assumption of one extra zero in  $(M_d)_{11}$  gives the famous relation[17, 18]  $V_{us} = \sqrt{m_d/m_s}$  which is seen then to be rather generic to a large class of models and ansätze. From there one can add more constraints, for example we have presented a new ansatz which can be made to fit the data quite well.

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